

Learning Spatially Regularized Correlation Filters for Visual Tracking

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SRDCF

- **ICCV 2015** paper
 - Poster session 4B on Wednesday afternoon
- **Won** the “OpenCV State of The Art Vision Challenge in Tracking”
- **Best** results in ICCV 2015 on OTB-2013
- **Competitive** results on VOT2015 and VOT-TIR2015
- [Webpage and Matlab code](#)

Discriminative Correlation Filters (DCF)



Standard DCF Formulation

$$S_f(x) = \sum_{l=1}^d x^l * f^l$$

Standard DCF Formulation



$$\downarrow$$
$$S_f(x) = \sum_{l=1}^d x^l * f^l$$

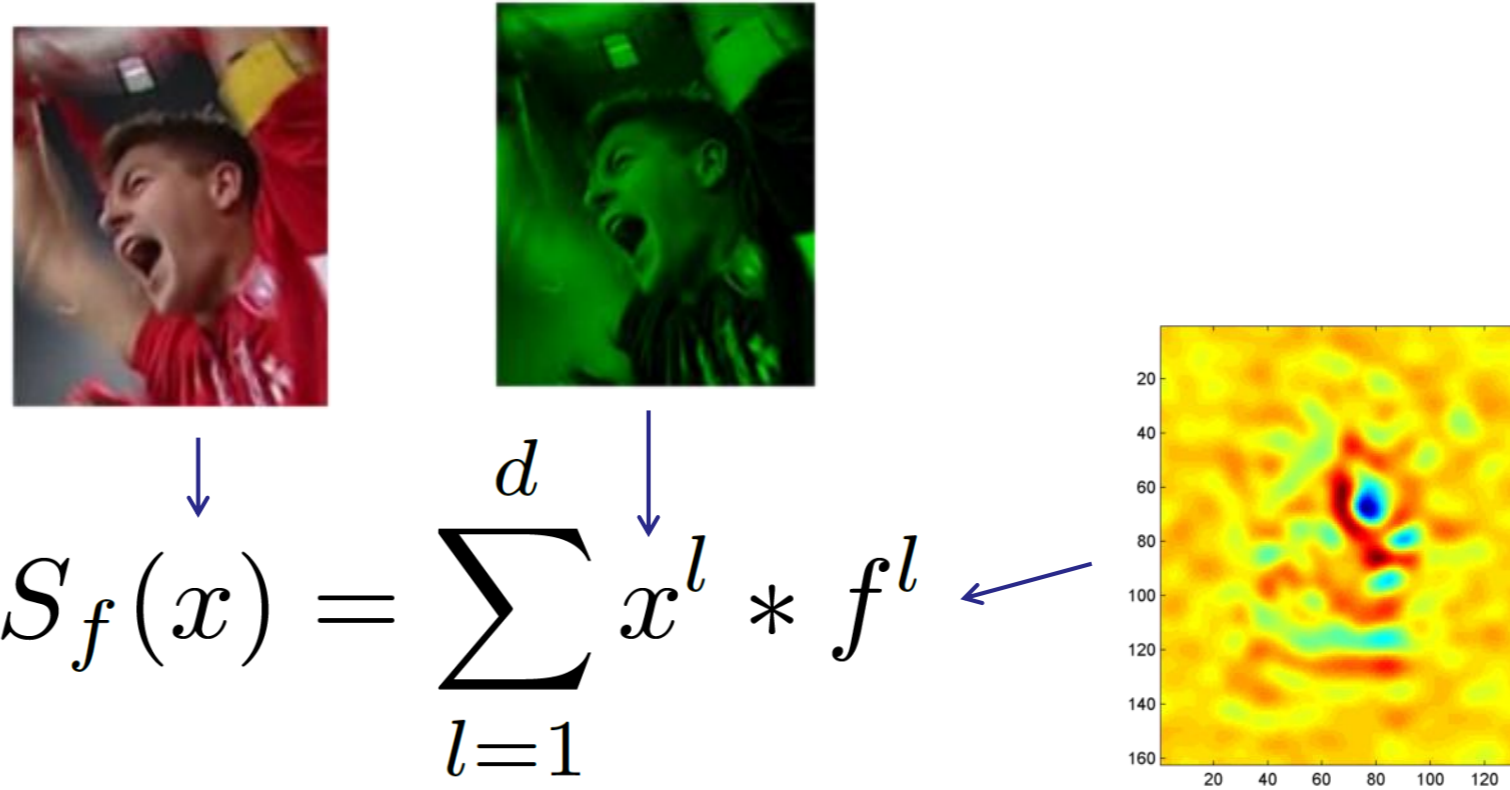
Standard DCF Formulation



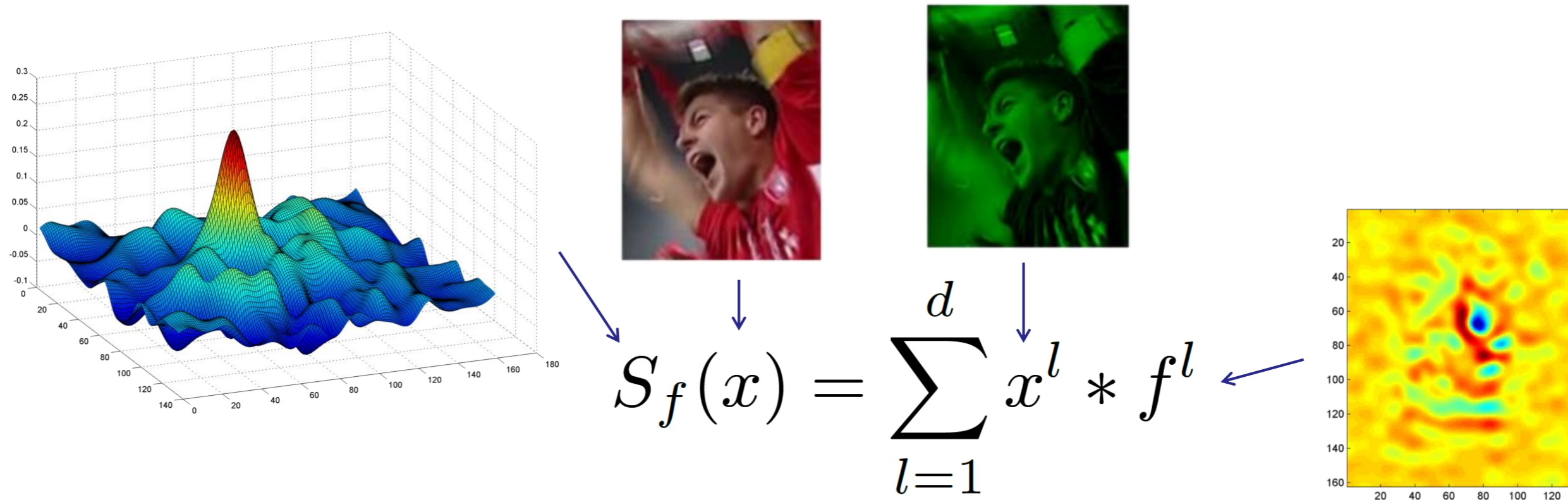
$$S_f(x) = \sum_{l=1}^d x^l * f^l$$

The equation shows the relationship between the original image and its decomposition. A blue arrow points from the original image to the variable x in the equation. Another blue arrow points from the green-tinted image to the variable f^l in the equation. The summation is over l from 1 to d .

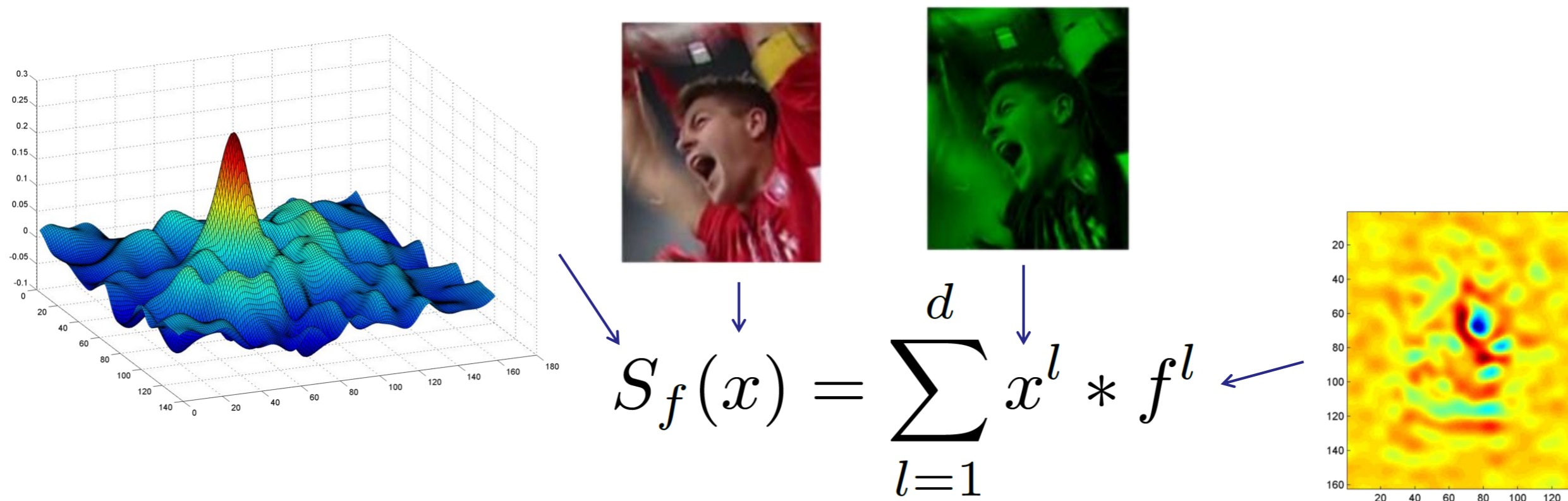
Standard DCF Formulation



Standard DCF Formulation

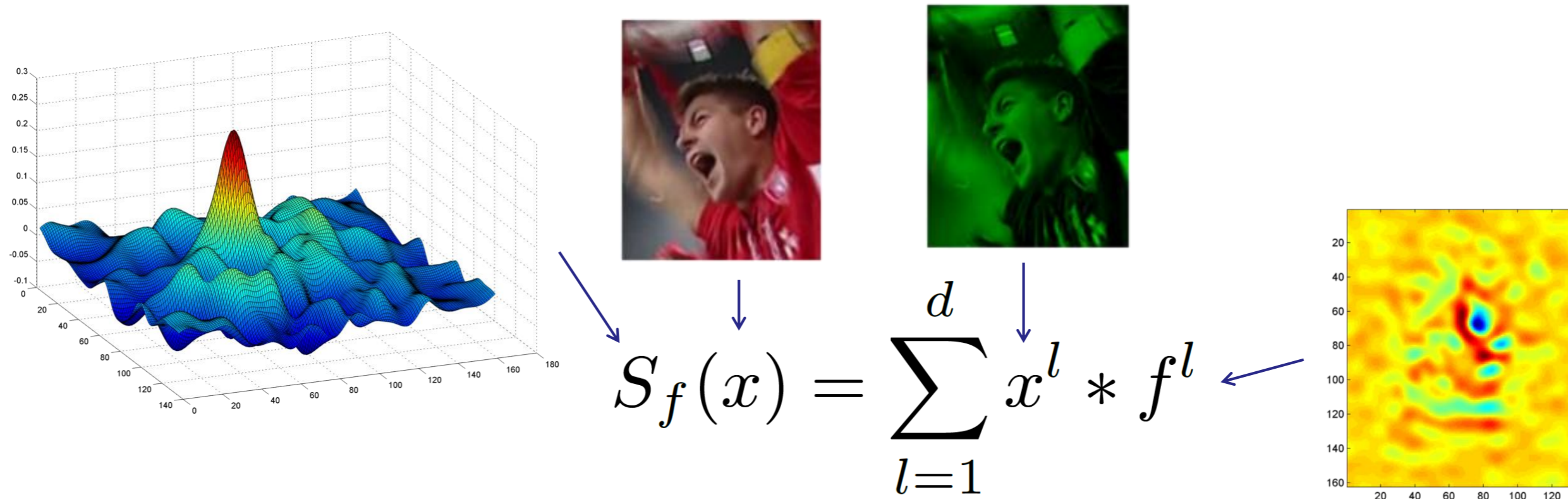


Standard DCF Formulation



$$\varepsilon_t(f) = \sum_{k=1}^t \alpha_k \left\| S_f(x_k) - y_k \right\|^2 + \lambda \sum_{l=1}^d \left\| f^l \right\|^2$$

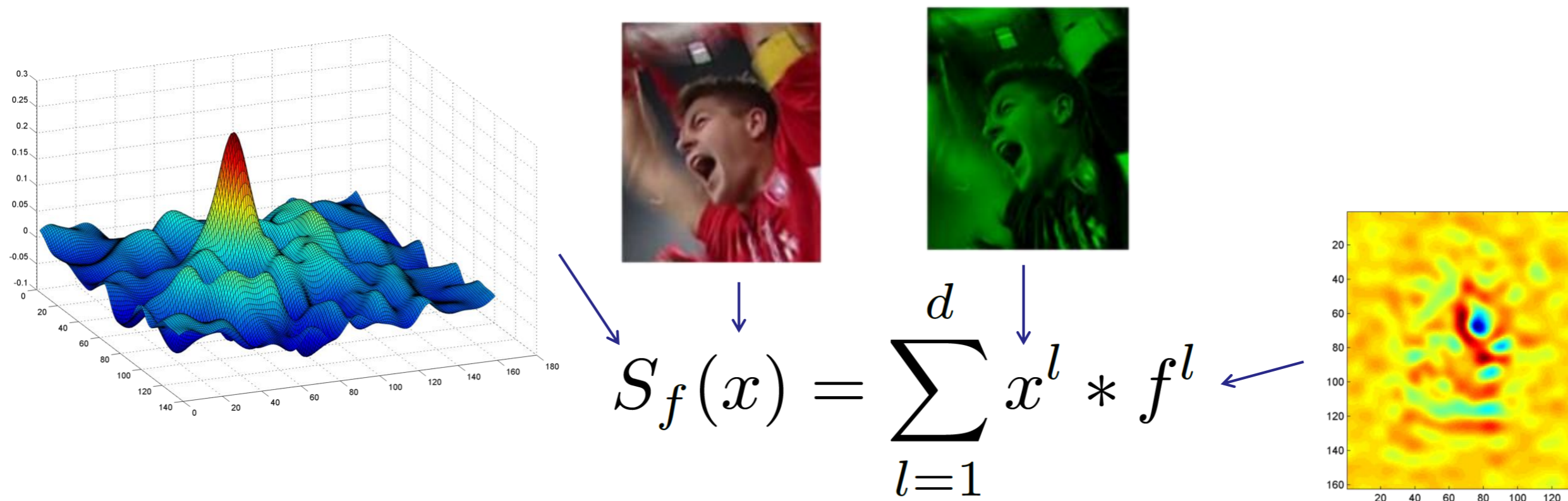
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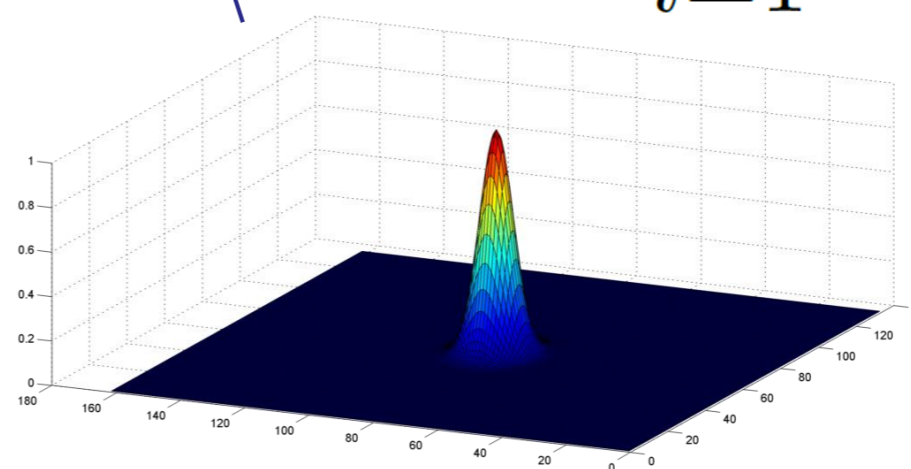
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Standard DCF Formulation



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Standard DCF Formulation

- Linear least squares problem
- Diagonalizable by the DFT if $d = 1$
- Incremental update schemes
 - Henriques et al. (ECCV 2012)
 - Danelljan et al. (CVPR 2014)
 - Danelljan et al. (BMVC 2014)
- Based on **harsh approximations**

Standard DCF Formulation

...has a major flaw...

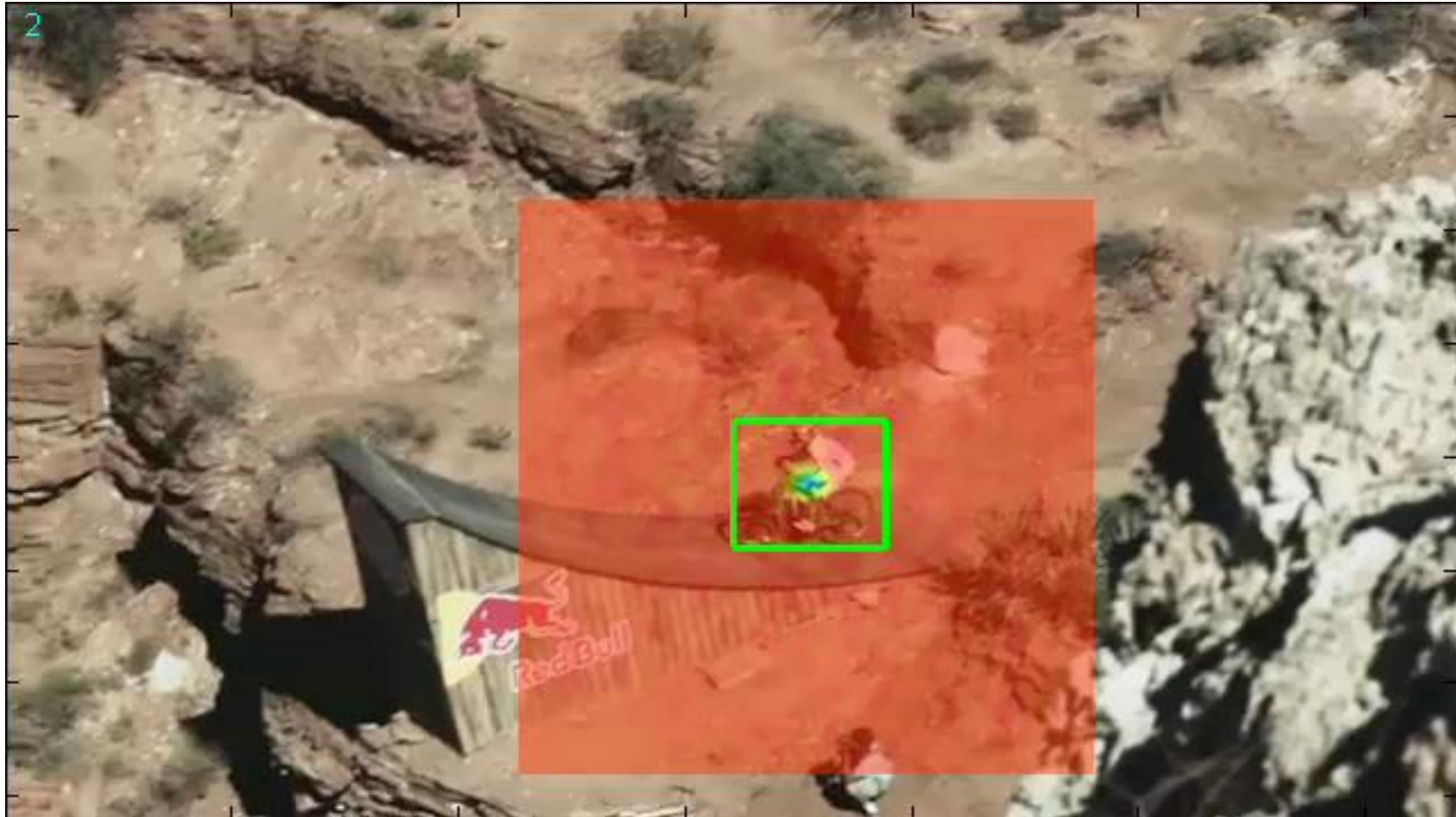
Circular Convolution \Leftrightarrow Periodic Extension



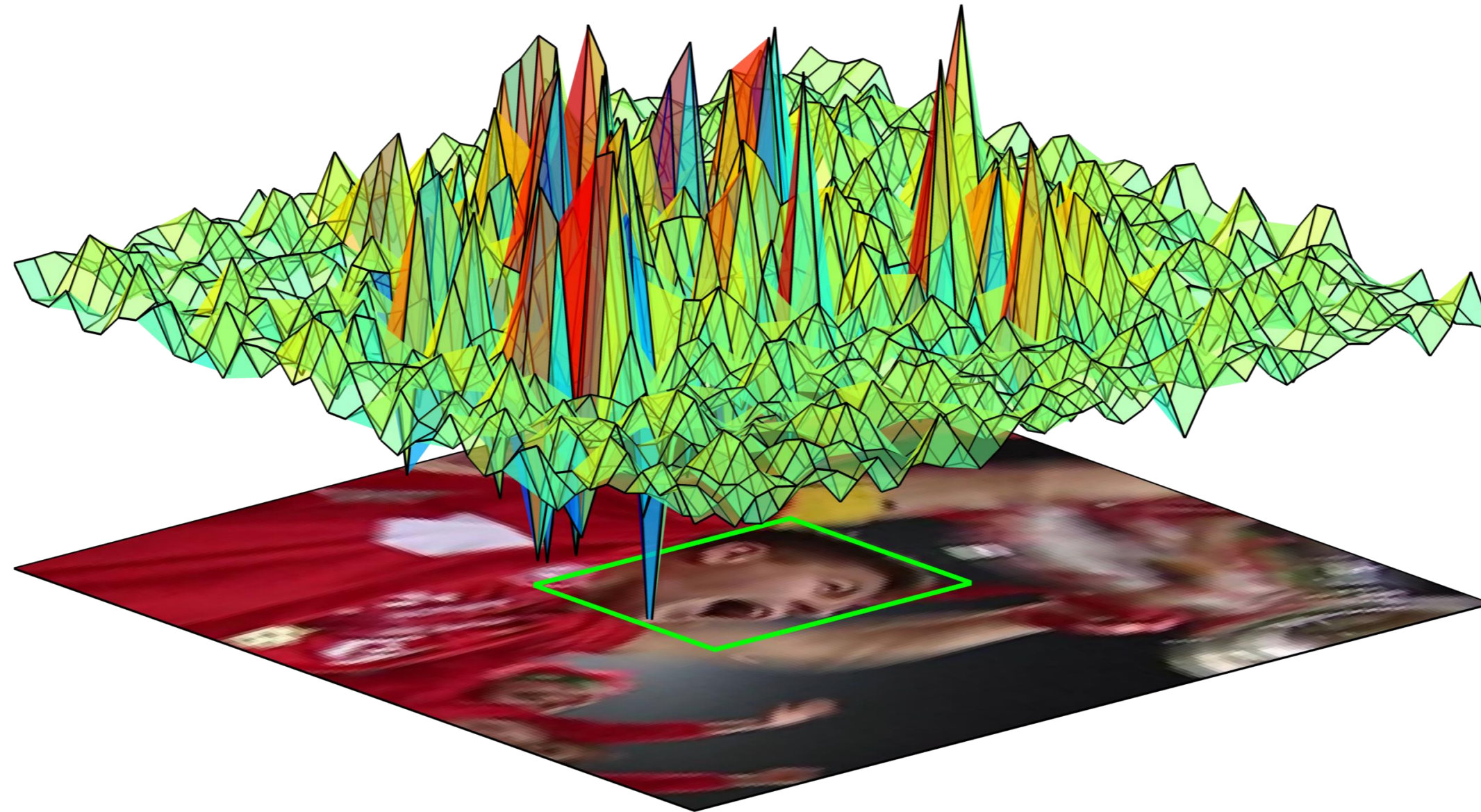
Larger Samples?



Larger Samples?



Why?



Limitations of a Small Sample Size

Forces a **small** sample size in training/detection

- Limited search region
- Limited training data
- Corrupted samples

Previous Work

- Optimizing a constraint filter
 - Fernandez et al. (PAMI 2015)
 - Galoogahi et al. (CVPR 2015)
- Leads to iterative optimization
 - Transition between spatial and Fourier domain
- Our approach:
 - More flexible
 - Efficient optimization in the Fourier domain

Our Approach: SRDCF

$$\varepsilon_t(f) = \sum_{k=1}^t \alpha_k \left\| \mathcal{S}_f(x_k) - y_k \right\|^2 + \lambda \sum_{l=1}^d \left\| f^l \right\|^2$$

Our Approach: SRDCF

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$$\varepsilon_t(f) = \sum_{k=1}^t \alpha_k \left\| S_f(x_k) - y_k \right\|^2 + \sum_{l=1}^d \left\| w \cdot f^l \right\|^2$$

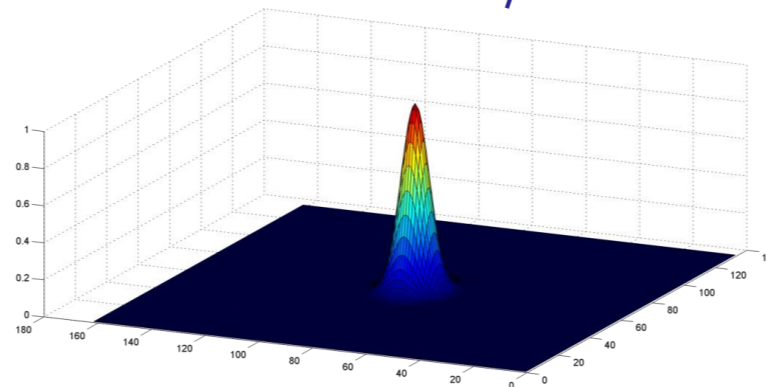
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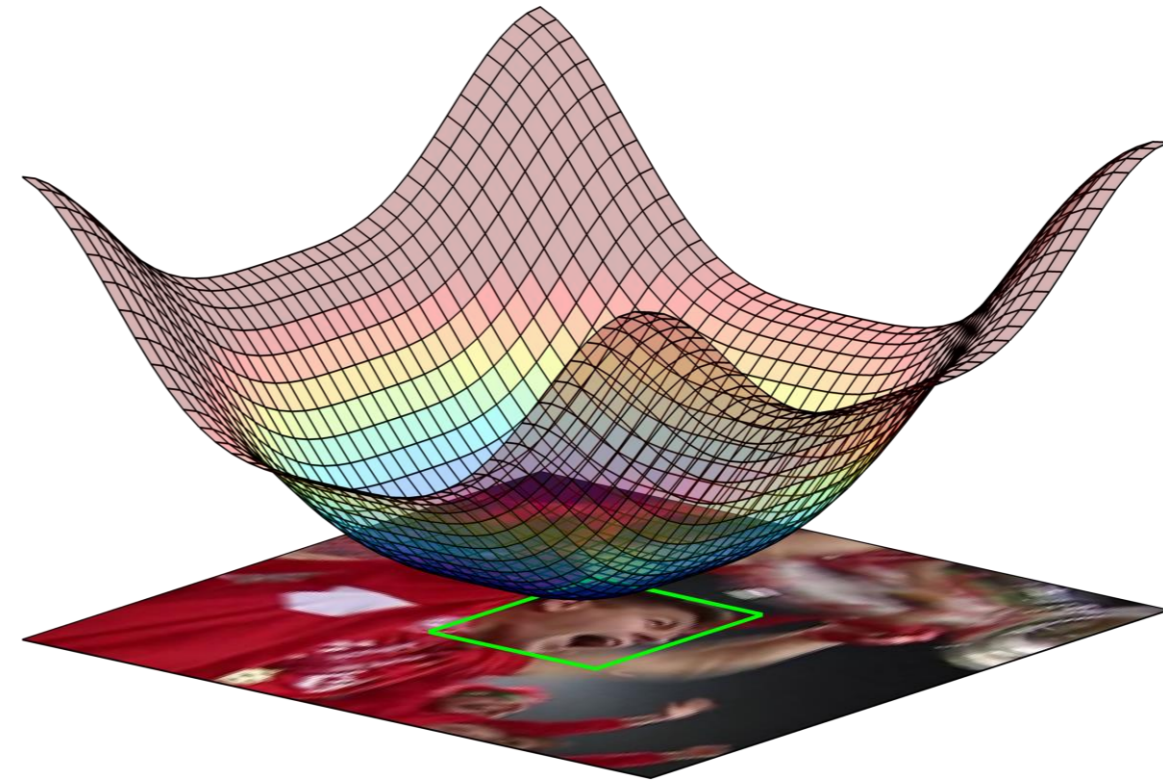
Our Approach: SRDCF



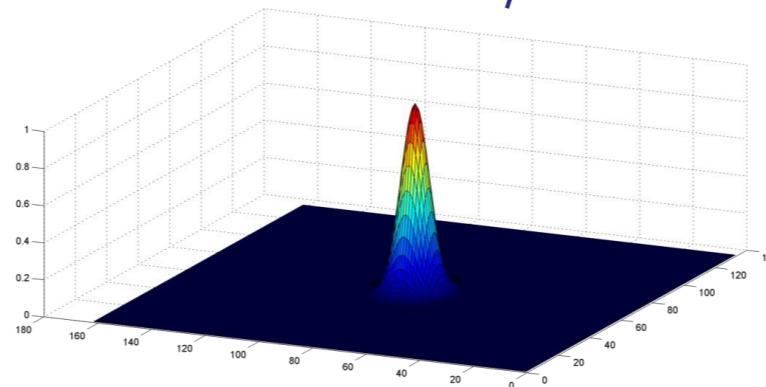
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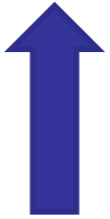


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DFT 

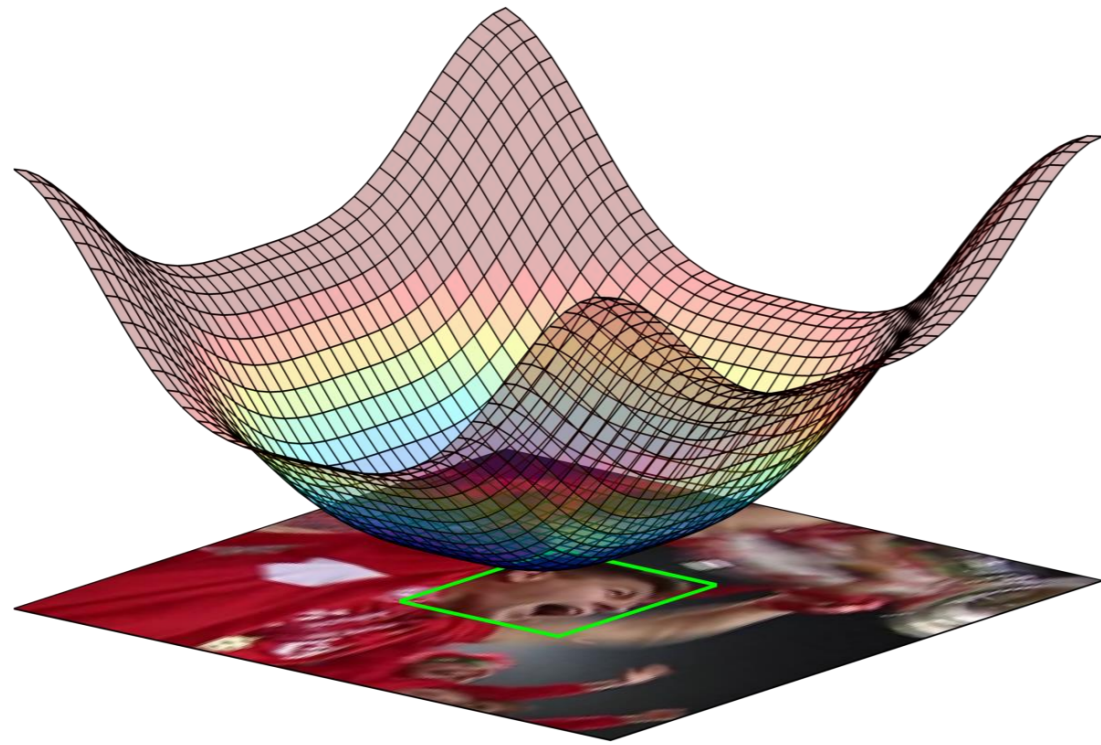
Our Approach: SRDCF

$$\check{\varepsilon}_t(\hat{f}) = \sum_{k=1}^t \alpha_k \left\| \sum_{l=1}^d \hat{x}_k^l \cdot \hat{f}^l - \hat{y}_k \right\|^2 + \sum_{l=1}^d \left\| \frac{\hat{w}}{MN} * \hat{f}^l \right\|^2$$

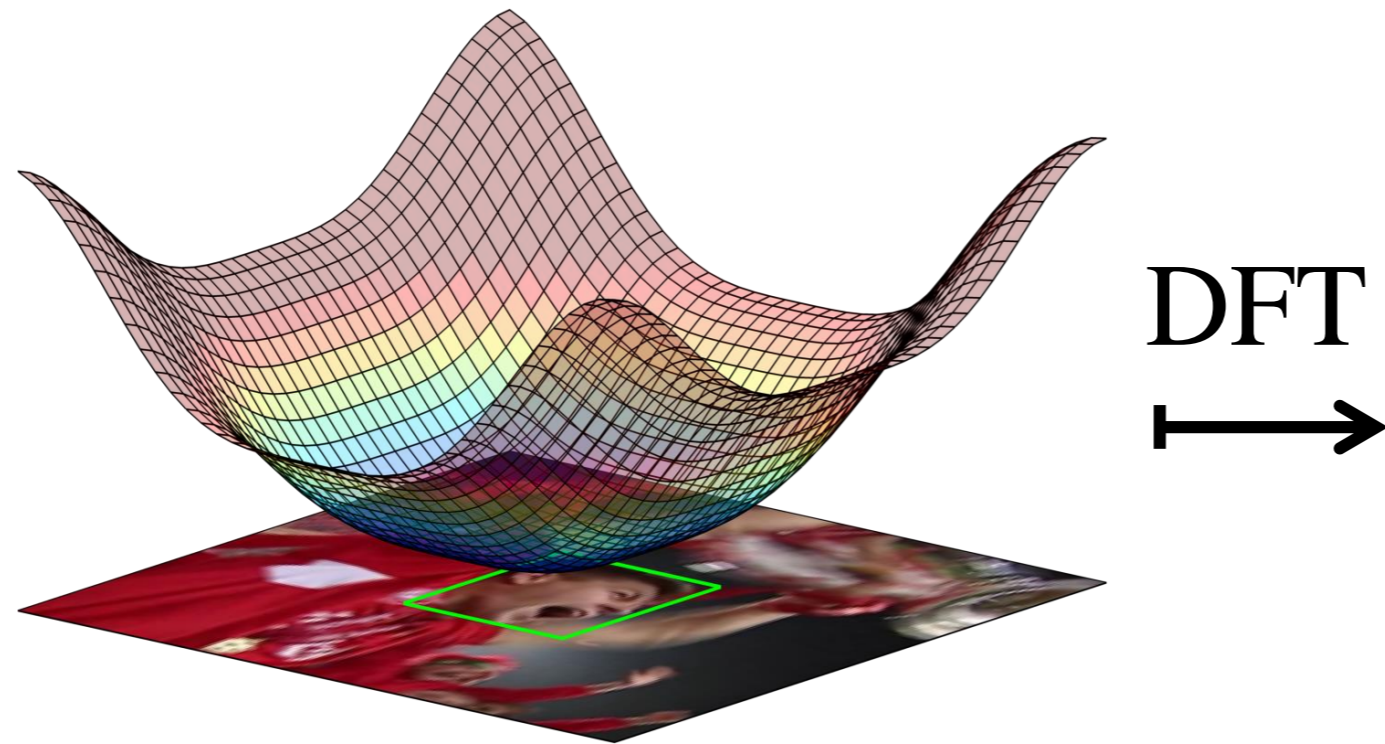
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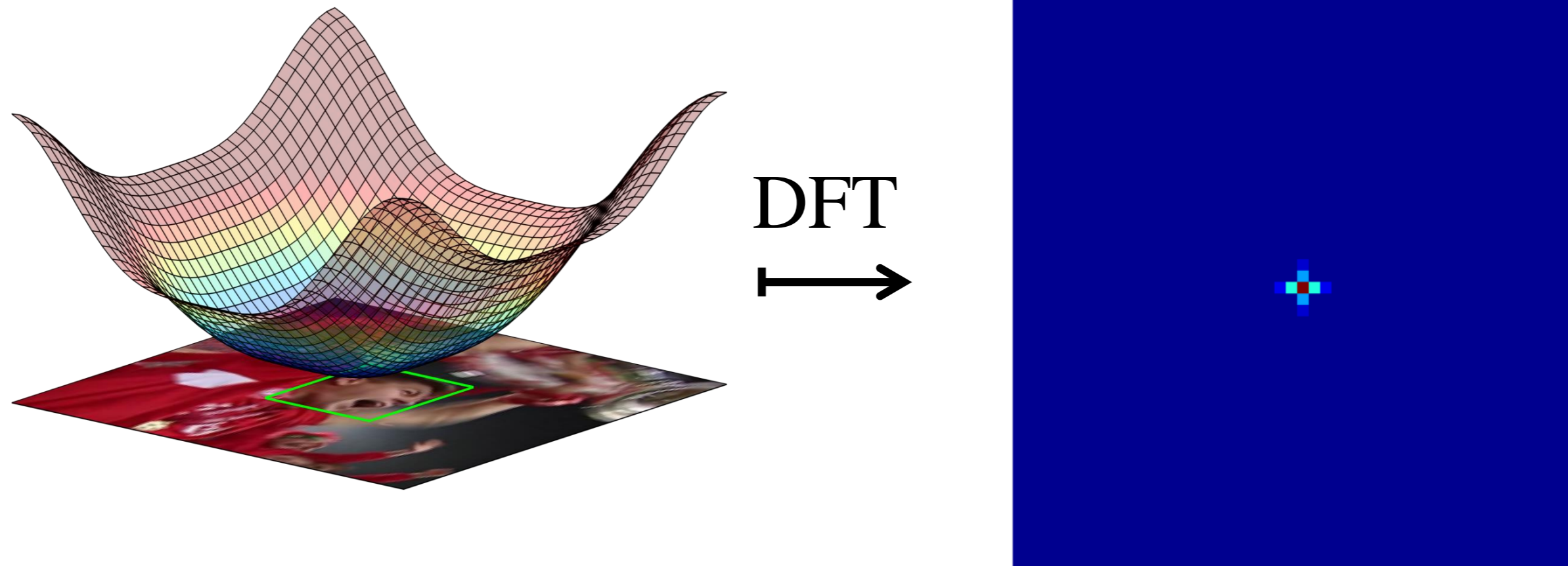
Our Approach: SRDCF



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$$\check{\mathcal{E}}_t(\hat{f}) = \sum_{k=1}^t \alpha_k \left\| \sum_{l=1}^d \hat{x}_k^l \cdot \hat{f}^l - \hat{y}_k \right\|^2 + \sum_{l=1}^d \left\| \frac{\hat{w}}{MN} * \hat{f}^l \right\|^2$$

Our Approach: SRDCF

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$$\hat{\mathbf{x}}_k^l, \hat{\mathbf{f}}^l, \hat{\mathbf{y}} \in \mathbb{C}^{MN}$$

Our Approach: SRDCF

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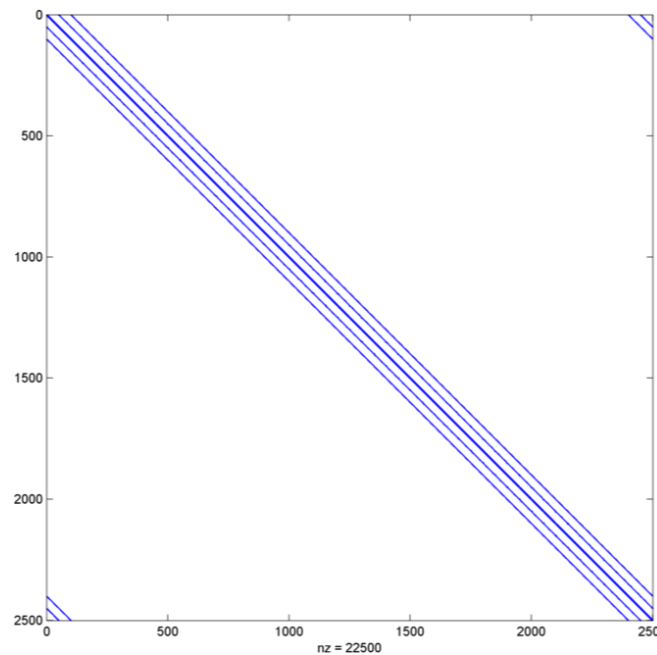
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$$\tilde{\mathbf{x}} = B\hat{\mathbf{x}} \in \mathbb{R}^{MN}$$

Our Approach: SRDCF

$$\check{\epsilon}_t(\hat{f}) = \sum_{k=1}^t \alpha_k \left\| \sum_{l=1}^d \hat{x}_k^l \cdot \hat{f}^l - \hat{y}_k \right\|^2 + \sum_{l=1}^d \left\| \frac{\hat{w}}{MN} * \hat{f}^l \right\|^2$$



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$$\hat{x}_k^l, \hat{f}^l, \hat{y} \in \mathbb{C}^{MN}$$



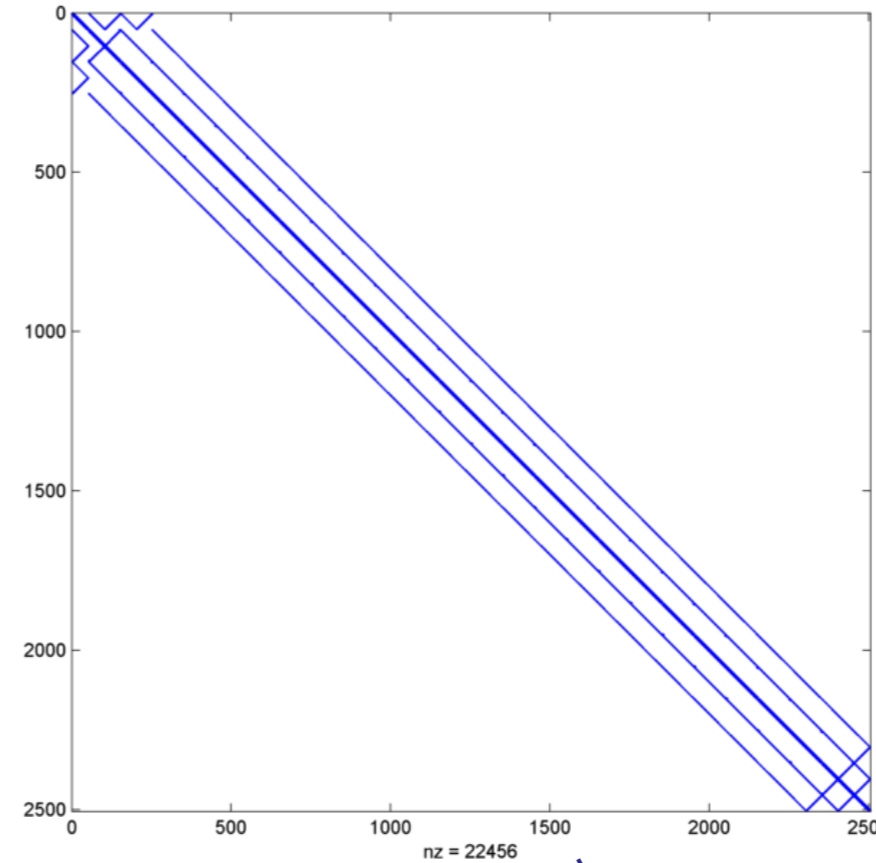
$$\tilde{\epsilon}(\tilde{\mathbf{f}}^1 \dots \tilde{\mathbf{f}}^d) = \sum_{k=1}^t \alpha_k \left\| \sum_{l=1}^d D_k^l \tilde{\mathbf{f}}^l - \tilde{\mathbf{y}}_k \right\|^2 + \sum_{l=1}^d \left\| C \tilde{\mathbf{f}}^l \right\|^2$$

$$\tilde{\mathbf{x}} = B \hat{\mathbf{x}} \in \mathbb{R}^{MN}$$

Our Approach: SRDCF

$$\tilde{\varepsilon}(\tilde{\mathbf{f}}^1 \dots \tilde{\mathbf{f}}^d) = \sum_{k=1}^t \alpha_k \left\| \sum_{l=1}^d D_k^l \tilde{\mathbf{f}}^l - \tilde{\mathbf{y}}_k \right\|^2 + \sum_{l=1}^d \left\| C \tilde{\mathbf{f}}^l \right\|^2.$$

Our Approach: SRDCF



$$\tilde{\epsilon}(\tilde{\mathbf{f}}^1 \dots \tilde{\mathbf{f}}^d) = \sum_{k=1}^t \alpha_k \left\| \sum_{l=1}^d D_k^l \tilde{\mathbf{f}}^l - \tilde{\mathbf{y}}_k \right\|^2 + \sum_{l=1}^d \left\| C \tilde{\mathbf{f}}^l \right\|^2.$$

Our Approach: SRDCF

$$\tilde{\varepsilon}(\tilde{\mathbf{f}}) = \sum_{k=1}^t \alpha_k \left\| D_k \tilde{\mathbf{f}} - \tilde{\mathbf{y}}_k \right\|^2 + \left\| W \tilde{\mathbf{f}} \right\|^2$$

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$$\tilde{\varepsilon}(\tilde{\mathbf{f}}) = \sum_{k=1}^t \alpha_k \left\| D_k \tilde{\mathbf{f}} - \tilde{\mathbf{y}}_k \right\|^2 + \left\| W \tilde{\mathbf{f}} \right\|^2$$



$$A_t \tilde{\mathbf{f}} = \tilde{\mathbf{b}}_t$$

Our Approach: SRDCF

$$\tilde{\varepsilon}(\tilde{\mathbf{f}}) = \sum_{k=1}^t \alpha_k \left\| D_k \tilde{\mathbf{f}} - \tilde{\mathbf{y}}_k \right\|^2 + \left\| W \tilde{\mathbf{f}} \right\|^2$$



$$A_t \tilde{\mathbf{f}} = \tilde{\mathbf{b}}_t$$

$$A_t = \sum_{k=1}^t \alpha_k D_k^T D_k + W^T W$$

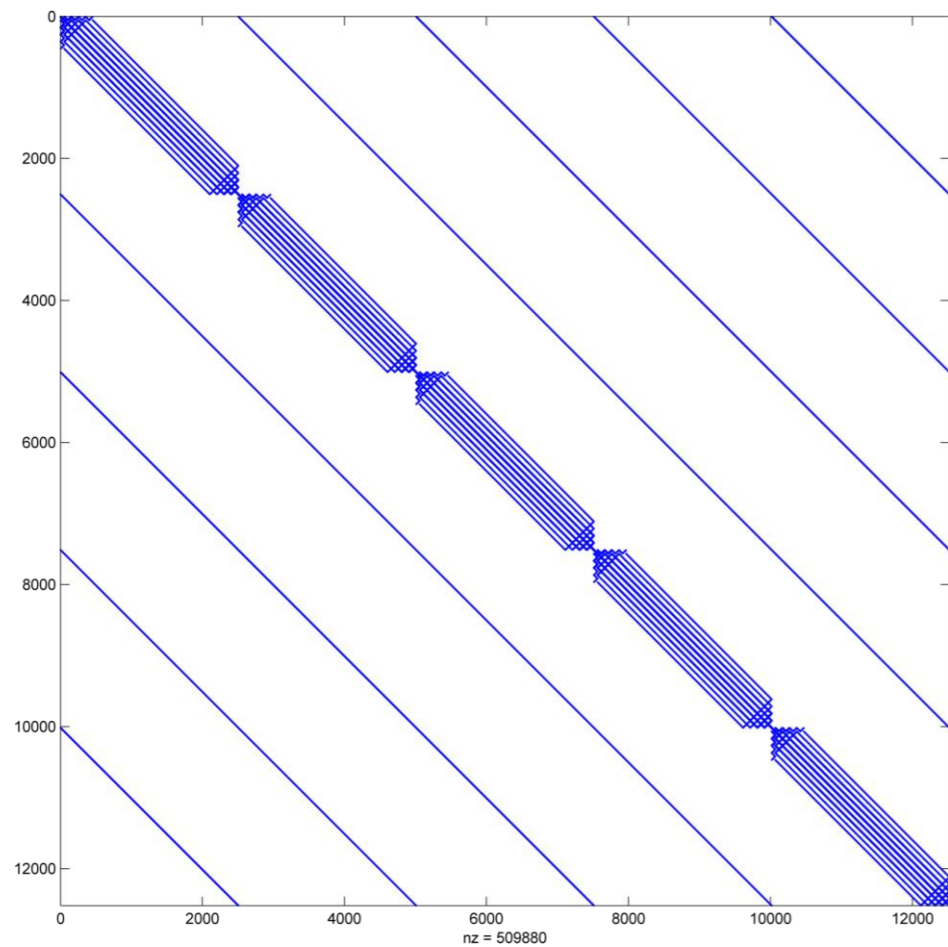
$$\tilde{\mathbf{b}}_t = \sum_{k=1}^t \alpha_k D_k^T \tilde{\mathbf{y}}_k.$$

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$$A_t = \sum_{k=1}^t \alpha_k D_k^T D_k + W^T W$$

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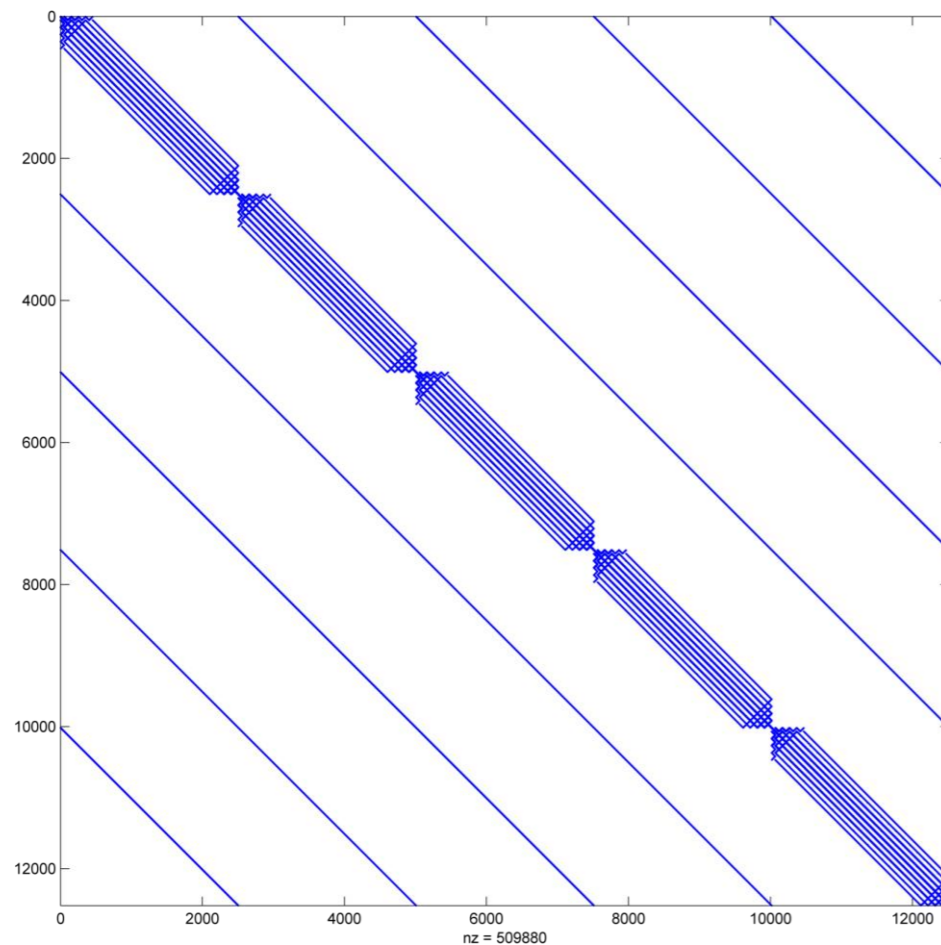
Our Approach: SRDCF



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Our Approach: SRDCF



$$\frac{2d + K^2}{dMN}$$

$$A_t = \sum_{k=1}^t \alpha_k D_k^T D_k + W^T W$$

$$\tilde{\mathbf{b}}_t = \sum_{k=1}^t \alpha_k D_k^T \tilde{\mathbf{y}}_k.$$

Incremental Update

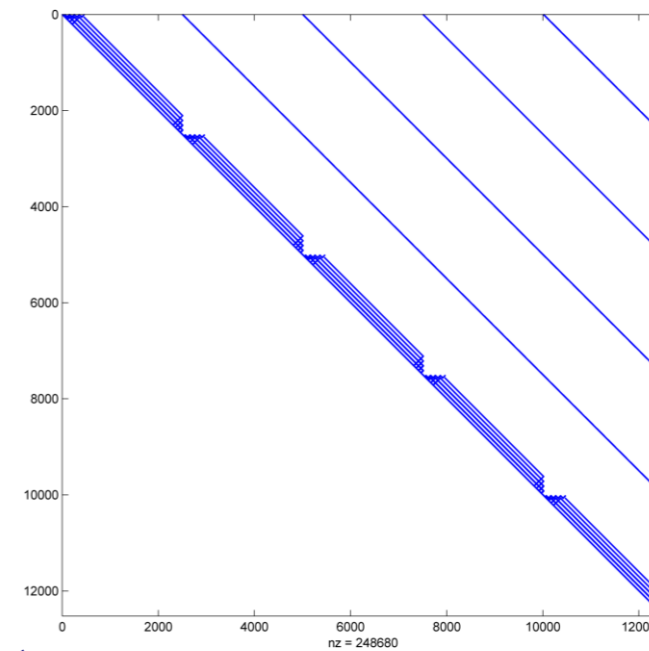
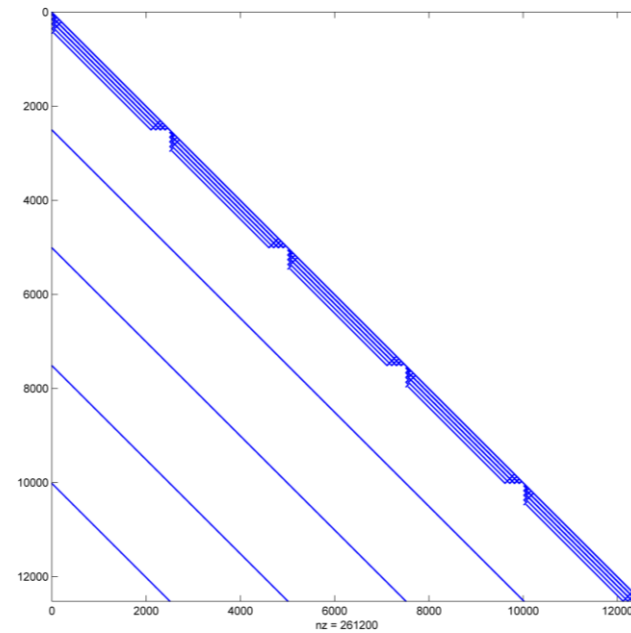
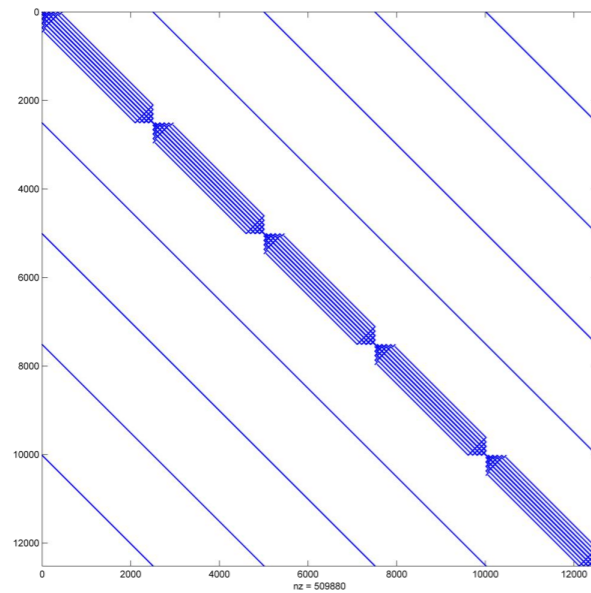
$$A_t = (1 - \gamma)A_{t-1} + \gamma (D_t^T D_t + W^T W)$$

$$\tilde{\mathbf{b}}_t = (1 - \gamma)\tilde{\mathbf{b}}_{t-1} + \gamma D_t^T \tilde{\mathbf{y}}_t.$$

Gauss-Seidel Optimization

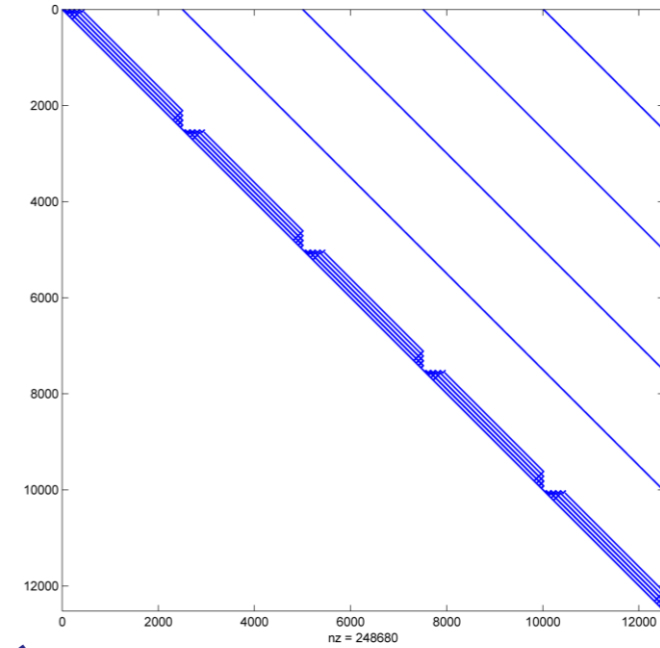
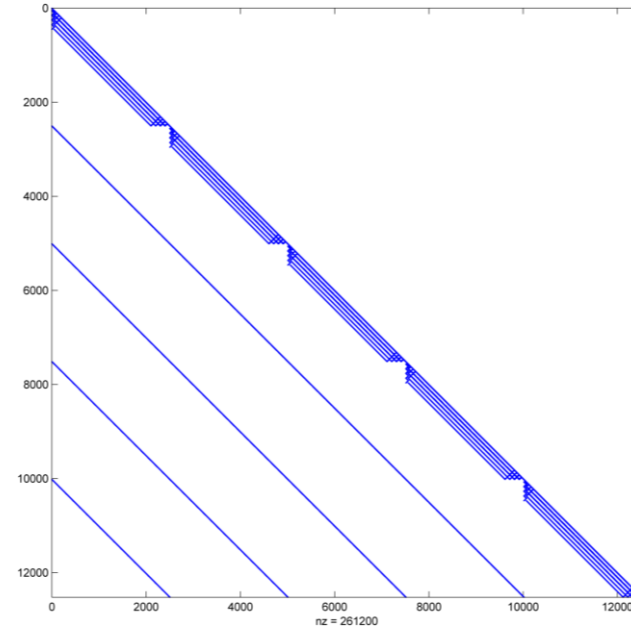
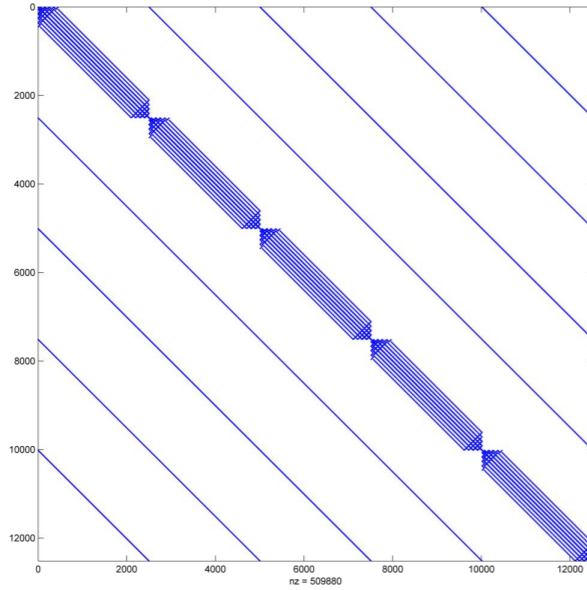
$$A_t = L_t + U_t$$

Gauss-Seidel Optimization



$$A_t = L_t + U_t$$

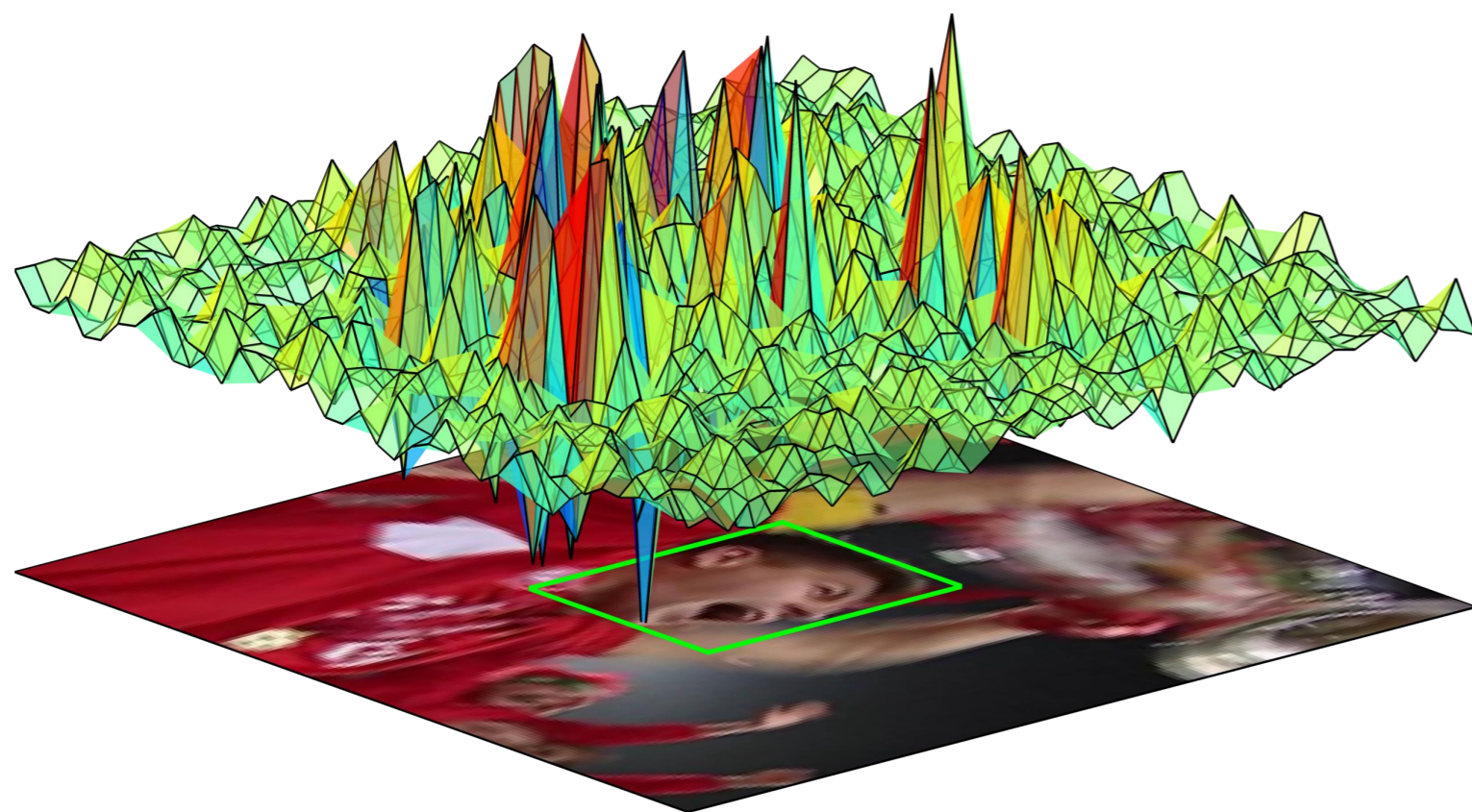
Gauss-Seidel Optimization



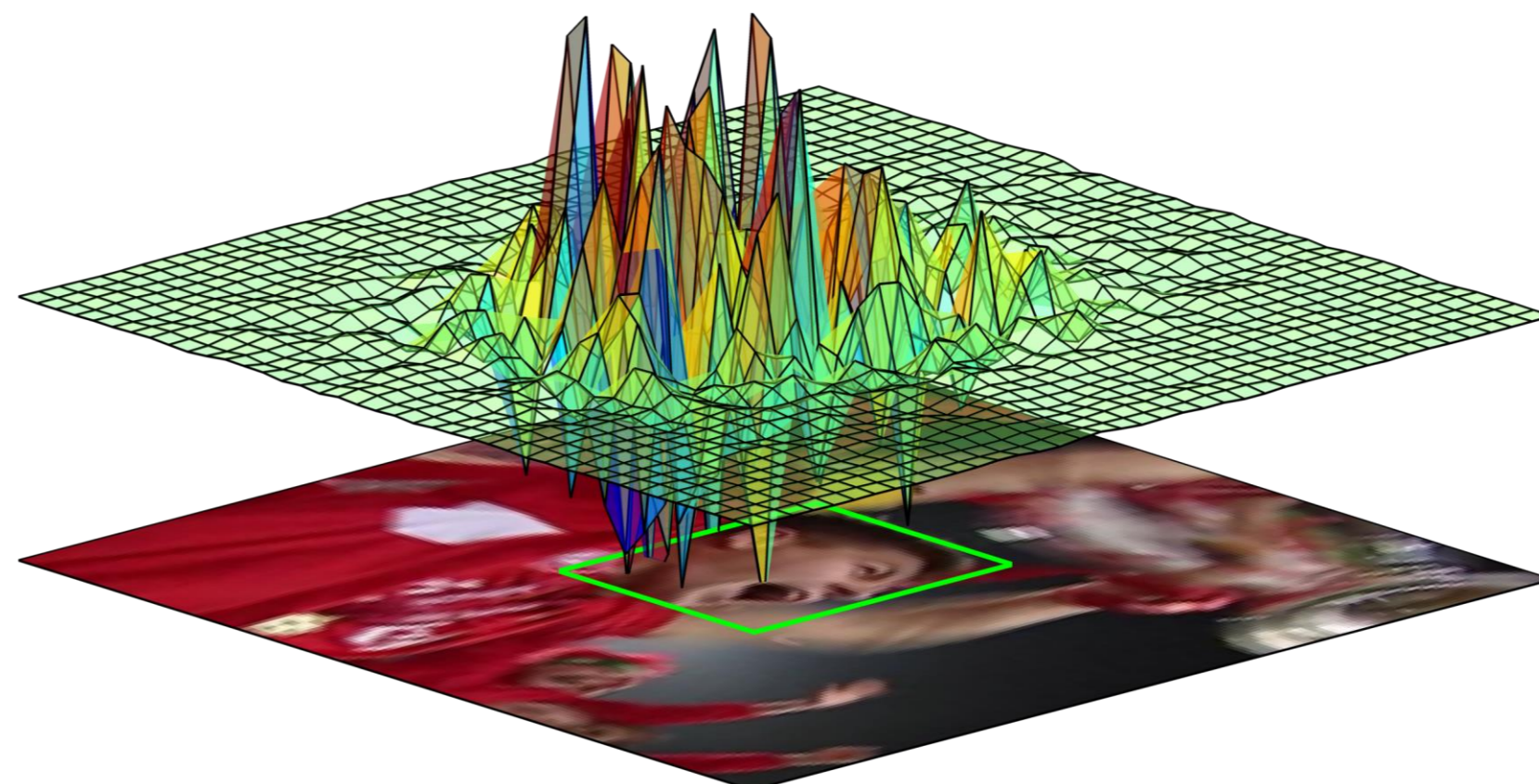
$$A_t = L_t + U_t$$

$$L_t \tilde{\mathbf{f}}^{(j)} = \tilde{\mathbf{b}}_t - U_t \tilde{\mathbf{f}}^{(j-1)}$$

Resulting Filter Coefficients

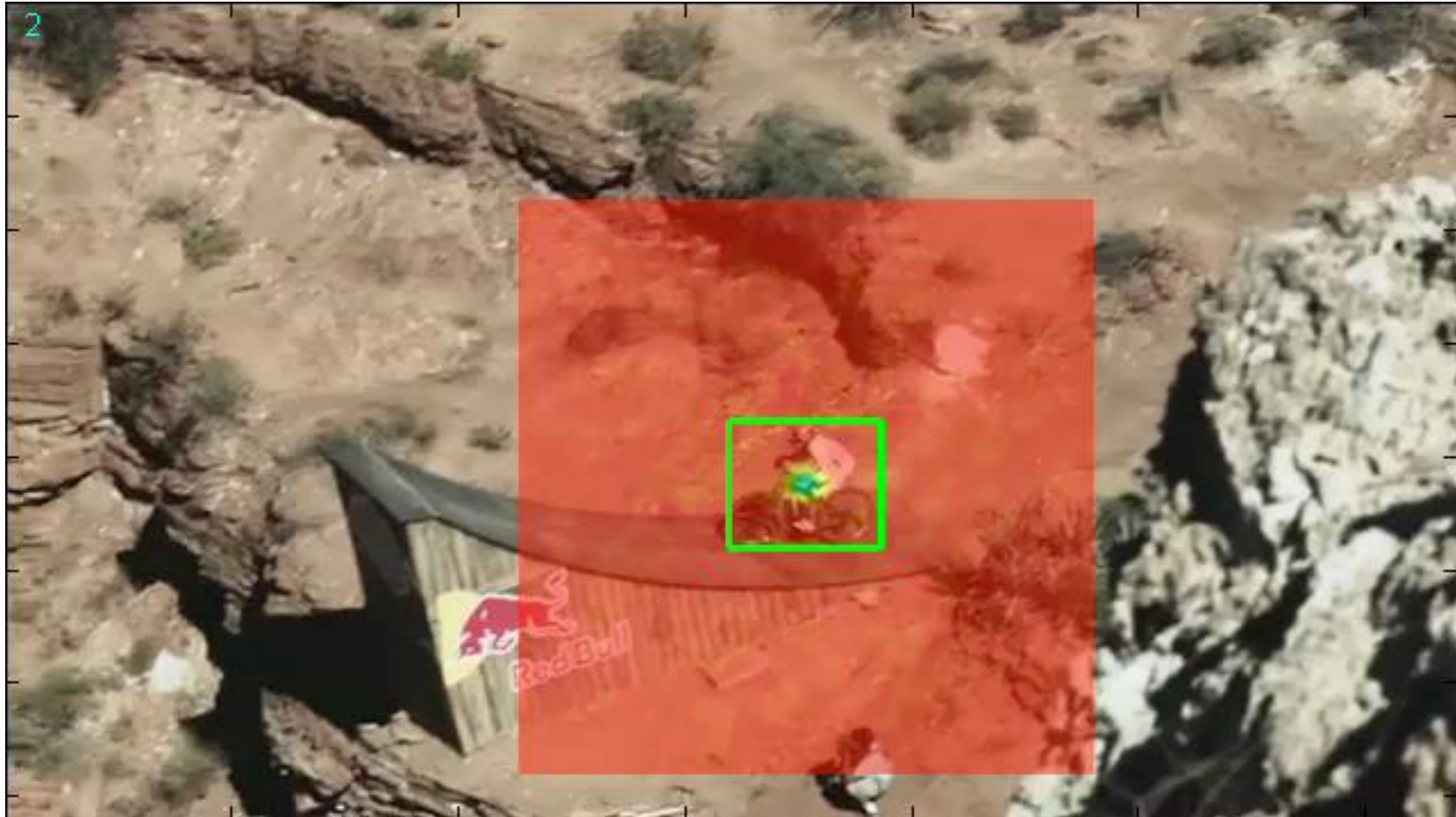


Standard DCF



Our SRDCF

Our Approach



Evaluation

- **Four benchmark datasets**
 - **OTB-2013**, Wu et al. (CVPR 2013)
 - **OTB-2015**, Wu et al. (PAMI 2015)
 - **ALOV++**, Smeulders et al. (PAMI 2014)
 - **VOT2014**
- **VOT2015** and **VOT-TIR2015**

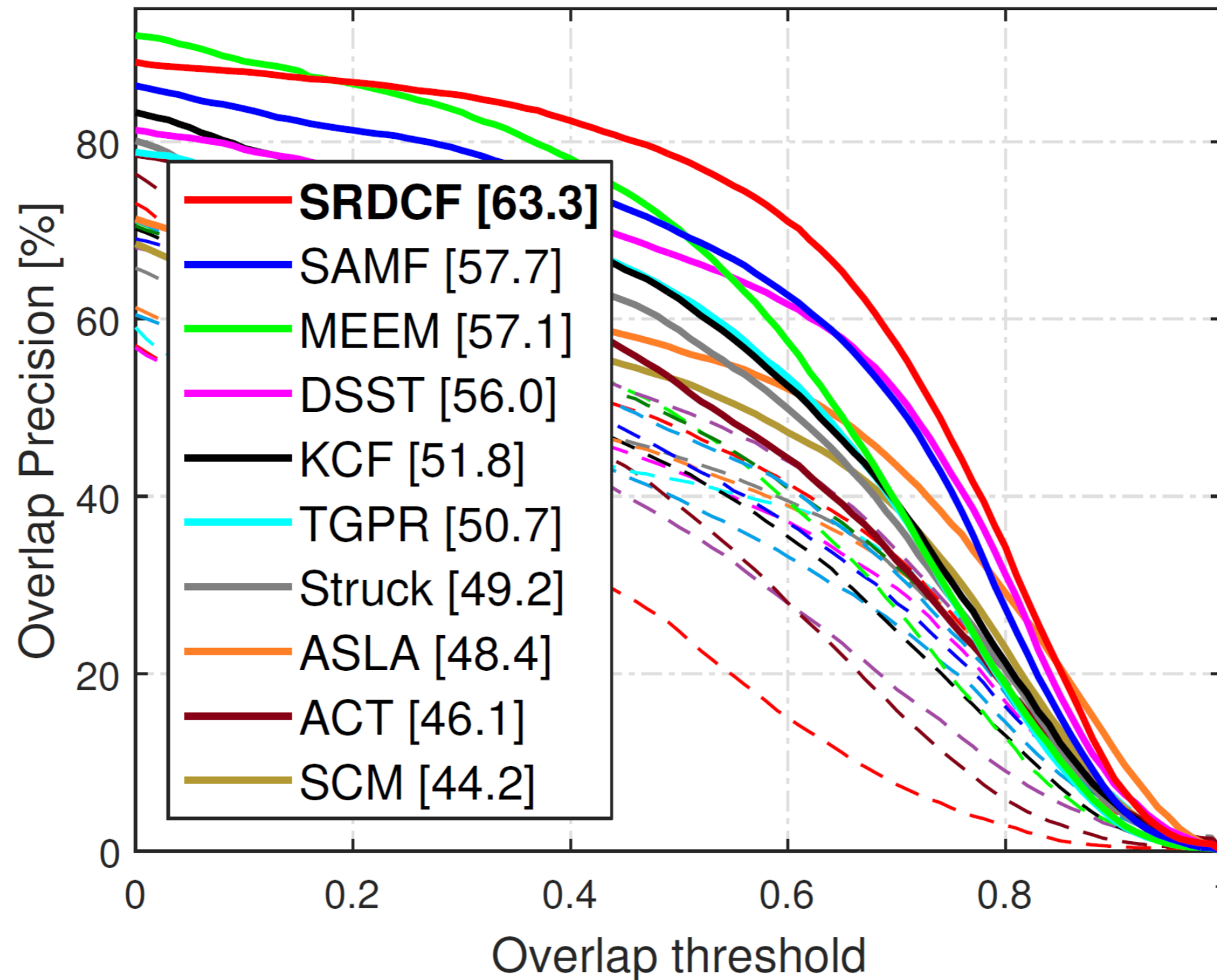
Impact of Regularization

- Baseline comparison on OTB-2013
 - Our framework
 - Using HOG features

	Conventional sample size		Expanded sample size	
Regularization	Standard	Ours	Standard	Ours
Mean OP (%)	71.1	72.2	50.1	78.1

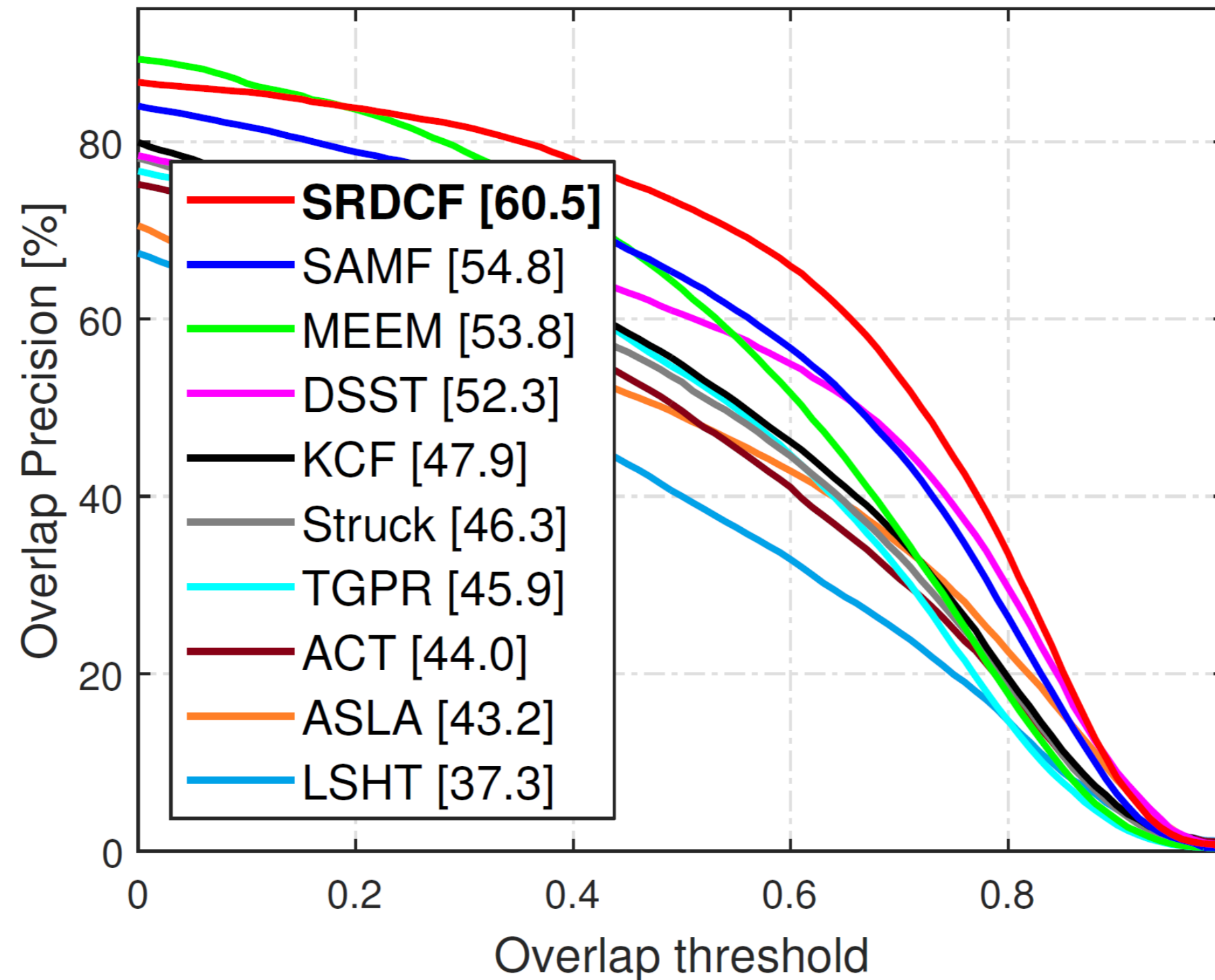
Online Tracking Benchmark 2013

Success plot on OTB-2013

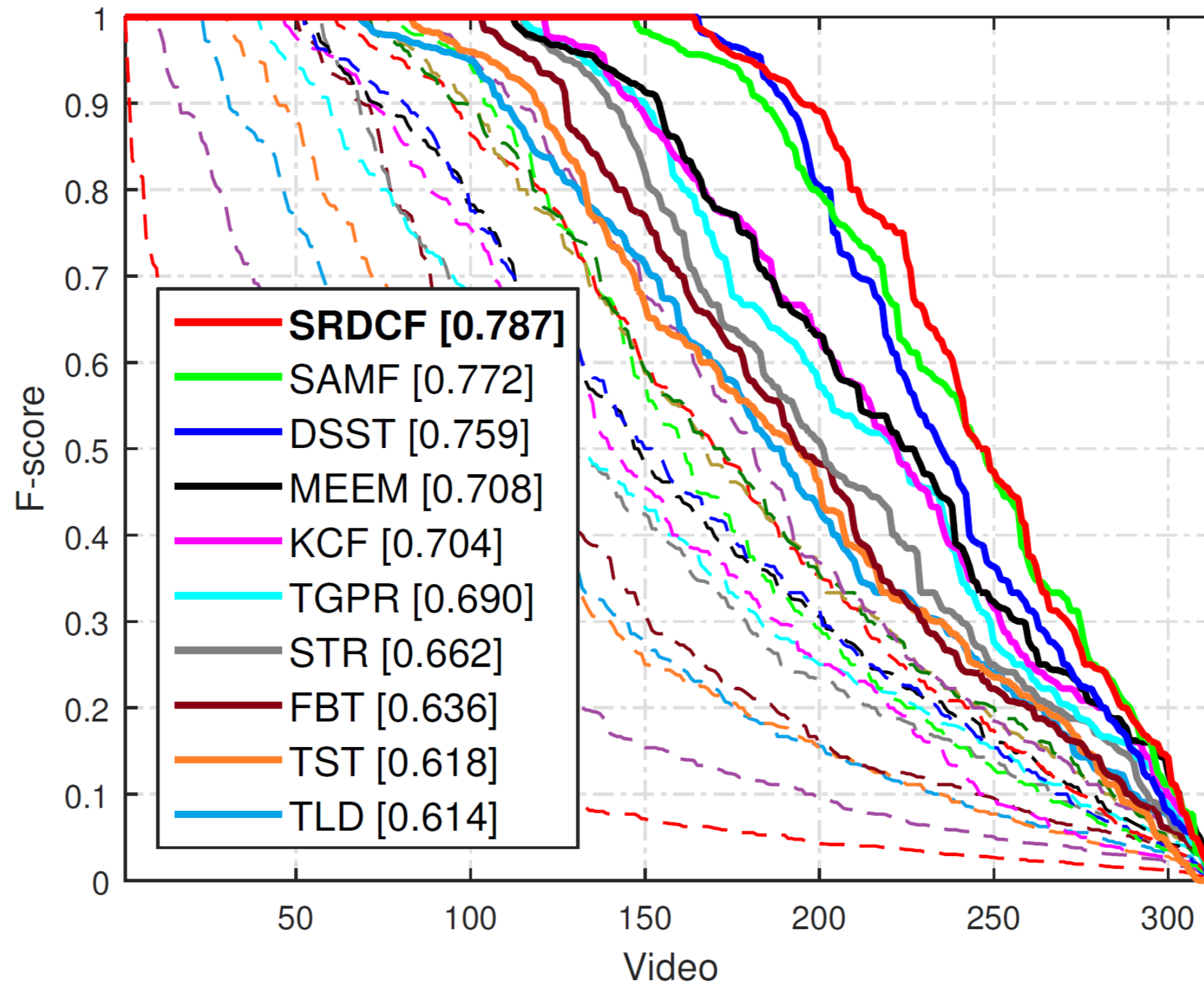


Online Tracking Benchmark 2015

Success plot on OTB-2015



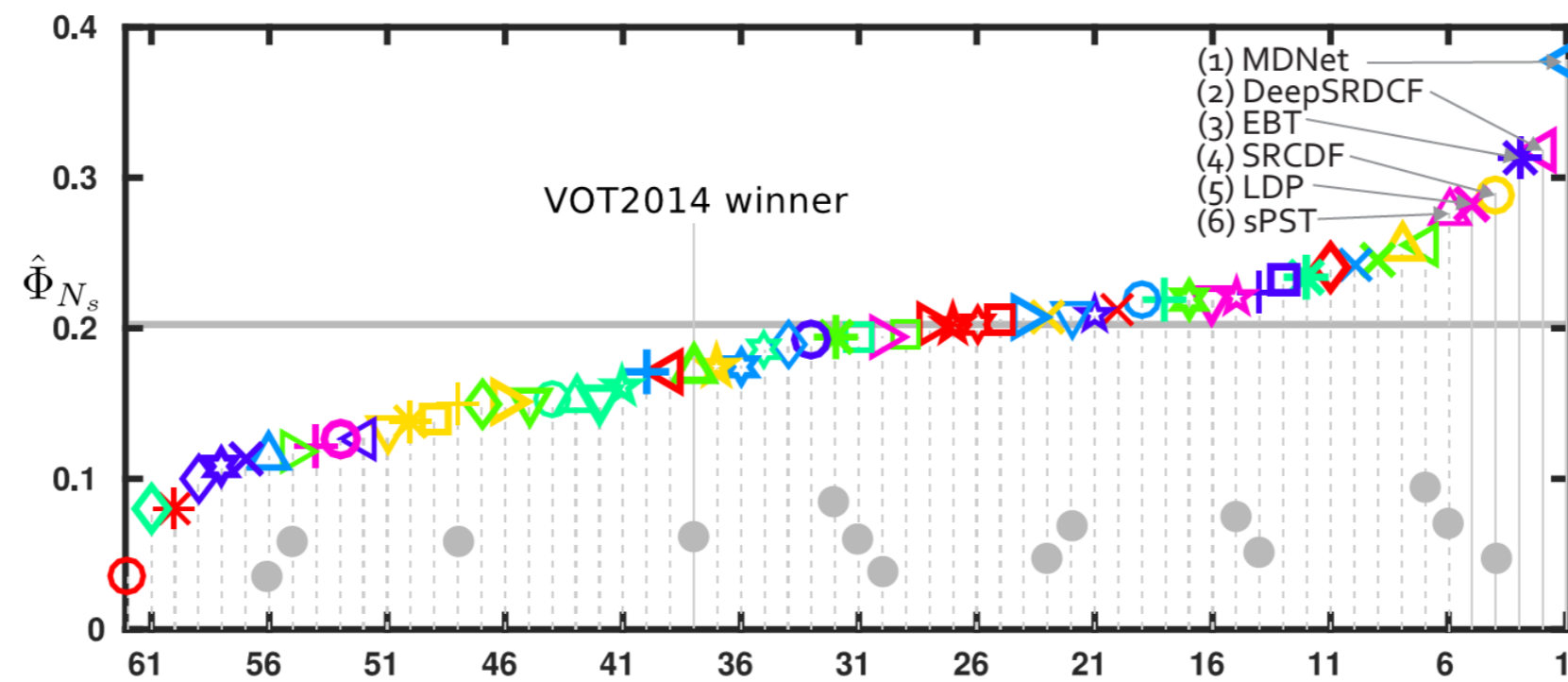
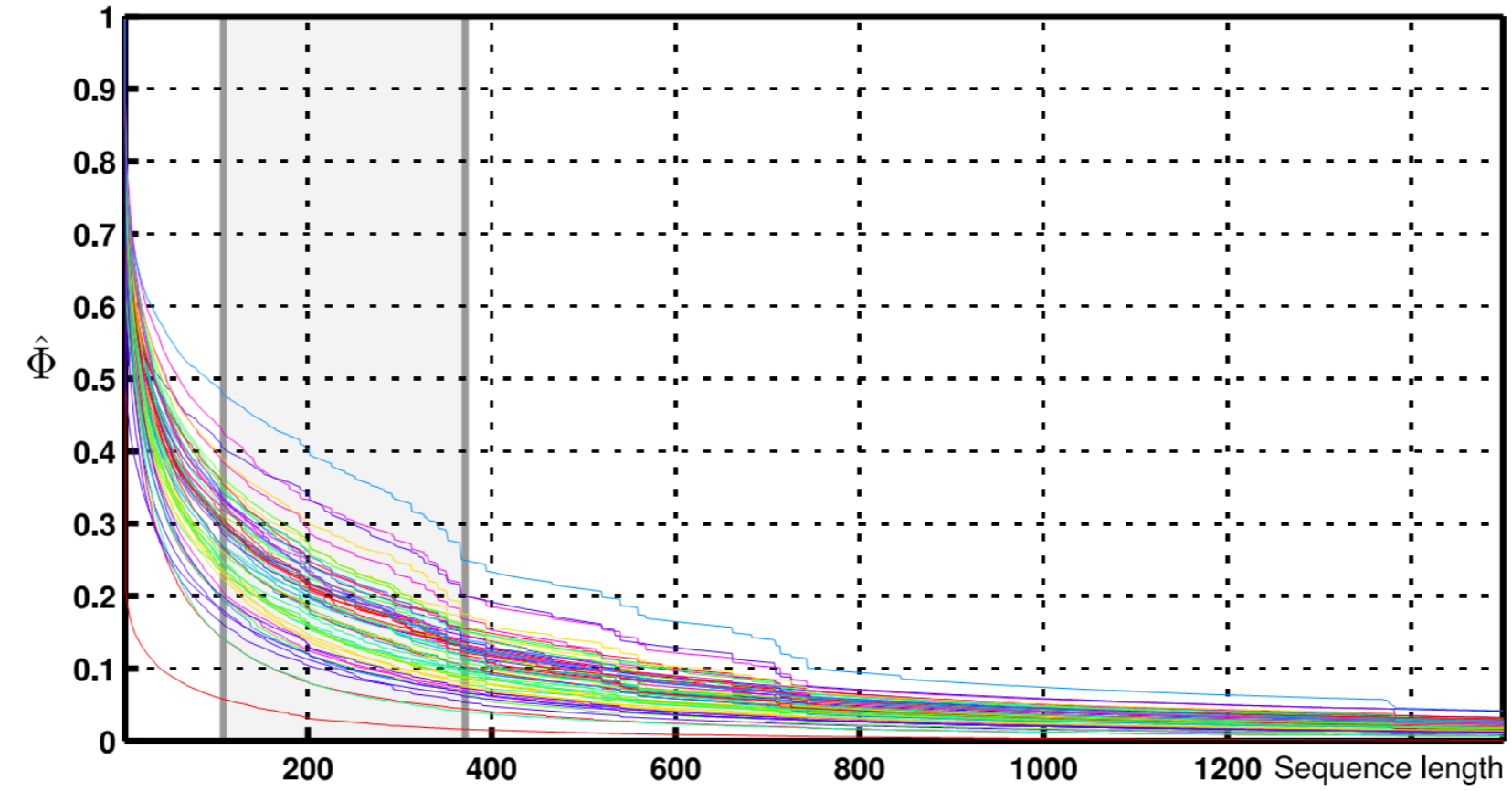
ALOV++ (314 videos)



ICCV 2015 Trackers on OTB-2013

Tracker	AUC (%)
SRDCF (ours)	63.3
SOWP (Han-UI Kim, Dae-Youn Lee, Jae-Young Sim, Chang-Su Kim)	61.9
Understanding and Diagnosing (Naiyan Wang, Jianping Shi, Dit-Yan Yeung, Jiaya Jia)	61.8
HCF (Chao Ma, Jia-Bin Huang, Xiaokang Yang, Ming-Hsuan Yang)	60.5
FCN (Lijun Wang, Wanli Ouyang, Xiaogang Wang, Huchuan Lu)	59.9
Proposal Selection (Yang Hua, Karteek Alahari, Cordelia Schmid)	58.0
TRIC-track (Xiaomeng Wang, Michel Valstar, Brais Martinez, Muhammad Haris Khan, Tony Pridmore)	53.0
LNL (Bo Ma, Hongwei Hu, Jianbing Shen, Yuping Zhang, Fatih Porikli)	50.8

VOT2015



VOT-TIR2015

- Modified features
 - HOG
 - Intensity channels
 - Thresholded frame difference

